

31.43. Visualize: Please refer to Figure P31.43.

Solve: Bulbs D and E are in series, so the same current will go through both and make them equally bright ($D = E$). Bulbs B and C are in parallel, so they have the same potential difference across them. Because they are identical bulbs with equal resistances, they will have equal currents and be equally bright ($B = C$). Now the equivalent resistance of B + C in parallel is less than the resistance of E, so the total resistance along the path through A is less than the total resistance along path through D. The two paths have the same total potential difference—the emf of the battery—so more current will flow through the A path than through the D path. Consequently, A will have more current than D and E and will be brighter than D and E ($A > D = E$). Bulbs B and C each have half the current of A, because the current splits at the junction, so A is also brighter than B and C ($A > B = C$).

The remaining issue is how B and C compare to D and E. Suppose B and C were replaced by wires with zero resistance, leaving just bulb A in the middle path. Then the resistance of the path through A would be half of the resistance of the path through D. This would mean that the current through A would be twice the current through D, so $I_A = 2I_D$. When B and C are present, their resistance adds to the resistance of A to lower the current through the middle path. So in reality, $I_A < 2I_D$. We already know that $I_B = I_C = \frac{1}{2}I_A$, so we can conclude that $I_B = I_C < I_D$. Since the current through B and C is less than the current through D and E, D and E are brighter than B and C. The final result of our analysis is $A > D = E > B = C$.